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WAVE FORCES ON SUBMERGED PIPELINES – A REVIEW WITH DESIGN AIDS

by D. A. Davis and J. B. Ciani

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INTRODUCTION

The design of submarine pipelines is of concern to the Naval Facilities Engineering Command (NAVFAC) because submarine pipelines are used at many Naval facilities for the shoreward transfer of fuel oil for power generation and seawater for desalination. Such pipelines are also used for sewage disposal at sea and for the protection of oceanographic cables.

These pipelines must be designed to resist the hydrodynamic forces imposed on them in the ocean environment. Waves have been known to severely damage pipelines in both shallow and deep water. For this reason, wave forces are of special concern, and the design of submarine pipelines to resist these forces is important.

Design aids for wave forces on submarine pipelines are required by NAVFAC design engineers, but these are not readily available. Methods for designing pipelines are not spelled out in NAVFAC design manuals; the validity of the usual techniques of pipeline design is frequently questioned. Furthermore, the coefficients used in this design must be empirically determined; agreement has not been reached on the values of these coefficients, although many tests have been performed in laboratories.

The objective of this study was to review and analyze published information and test results on wave-induced forces on submarine pipelines and to develop design aids for calculating these forces.

This report considers both horizontal and vertical forces on pipelines in the ocean; specifically, pipelines that are either on the bottom or above the bottom but are not partially or fully buried in the bottom. The pipelines are assumed to be rigidly anchored to the seafloor and are presumed not to move under the influence of passing waves. Although the pipelines are within the influence of surface waves, they are submerged sufficiently deep that they have no effect on the surface profile. The clearance of the pipeline above the bottom and the orientation of the pipeline relative to the wave fronts are considered. Breaking waves are not considered nor are wave forces on groups of pipelines.

The theoretical aspects of the interaction of water waves and submarine pipelines are reviewed. It is shown that the complex physics of the design problem defies a strictly theoretical approach at this time. Consequently, previous work directed toward developing semiempirical design data are reviewed. Design aids which use the results of this previous work are presented.

PROBLEM DEFINITION

Figure 1 is a definition sketch which will serve as a point of departure for the discussion of flow physics about horizontal cylinders. In the general case, the cylinder of diameter D is suspended some distance e above a plane boundary. Considerable interest is attached to the case where the cylinder rests on the boundary; that is, $e = 0$. The water depth is designated by d and the height and wavelength of the incident waves by

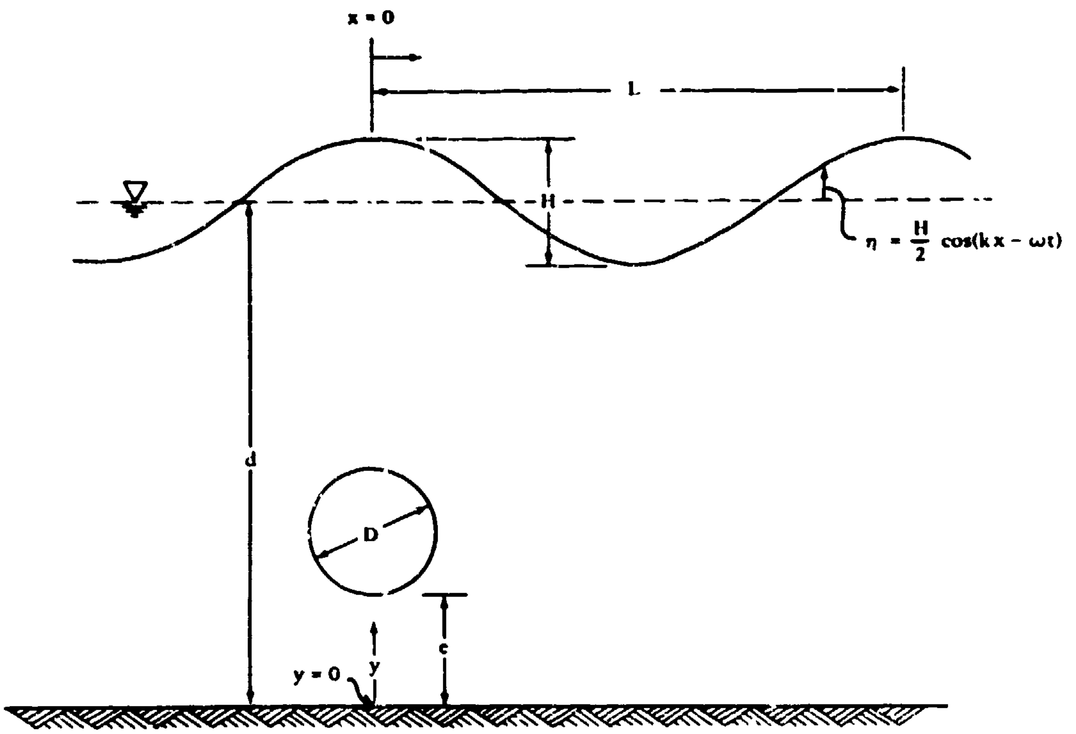

Figure 1. Definition sketch.

the symbols H and L, respectively. The local water surface elevation, a function of both time t and horizontal space coordinate x is given by η. Finally, it is noted that the origin of the vertical coordinate of interest y is on the plane boundary beneath the cylinder.

Forces on Cylinders in Nonoscillating Flow

Consider Figure 2a, which depicts a cylinder immersed in an infinite expanse of an inviscid fluid. Drag on the cylinder and flow separation along the cylinder due to fluid viscosity do not occur. The pressure on both the forward and rearward faces of the cylinder are the same, thus the drag is zero. Since vortex formation due to viscosity is absent, the lift force acting on the cylinder is also zero.

Now consider Figure 2b which shows a stationary circular cylinder in a uniform, steady stream of real (viscid) fluid. The streamlines near the cylinder initially follow the contour of the cylinder. Because of momentum loss within the shear layer next to the cylinder, the streamlines separate from the cylinder to form a wake. Two forms of cylinder drag are present. The first is due to fluid viscosity and the velocity gradient within the thin boundary layer next to the cylinder's surface. This drag component is termed the friction drag. However, the more important component of drag at higher Reynolds numbers N_R* is the form drag caused by pressure variation along the wall of

*$N_R = VD/\nu$, a dimensionless number where V is the free stream velocity, D is the cylinder diameter and ν is the fluid kinematic viscosity.

the cylinder resulting in a low pressure region in the wake. The drag force F_D per unit length of cylinder is represented by the following expression

$$F_D = \frac{1}{2} C_D \rho D v^2 \qquad (1)$$

where ρ is the mass density of the fluid, D is the cylinder diameter, v is the free stream velocity, and C_D is a nondimensional number designated the drag coefficient.

The votices which form in the cylinder wake give rise to a force normal to the drag force. This "lift" force F_{LE}, which is velocity dependent, acts alternately in the upward direction and the downward direction and has a mean magnitude of zero. The lift force can have an important effect on cylindrical structures if the natural frequency of the structure is near that of the vortex shedding frequency. A dimensionless vortex shedding frequency, the Strouhal number, is given by

$$S = \frac{n D}{v} \qquad (2)$$

where n is the frequency in Hertz. At Reynolds numbers between 10^3 and 10^5, the Strouhal number remains fairly constant at S = 0.21. Roshko (1961) has shown that the Strouhal number increases somewhat at higher Reynolds numbers, becoming as great as 0.27 as N_R approaches 10^7.*

A stationary cylinder immersed in an accelerating viscid flow field experiences an inertial force (Figure 2c). An expression for the inertial force per unit length of cylinder can be written as:

$$F_I = C_I \rho \frac{\pi D^2}{4} \dot{v} \qquad (3)$$

where $\dot{v}$ is the fluid acceleration and C_I is a nondimensional coefficient of inertia. By using potential flow theory, it can be shown that $C_I = 2.0$ for a cylinder in an accelerating stream of inviscid fluid. The inertial force is affected somewhat by the presence of the wake.

For cylinders resting on or near a plane boundary, some additional flow effects must be considered (see Figure 2d). A boundary layer is established near a bottom plane within which the horizontal water particle velocity varies from zero at the wall to the free stream velocity at some elevation above the wall. The boundary layer has an important effect on cylinder lift and drag. However, in tests made near a bottom boundary, no one has measured the boundary layer thickness and related it directly to the measured cylinder forces.

The flow asymmetry created when a cylinder is brought near a plane generates a lift force F_{LA} normal to the flow which is velocity dependent and which can be of considerable magnitude for small gaps. This lift force which acts in a downward direction should be distinguished from that due to the vortex formation F_{LE} which was discussed previously.

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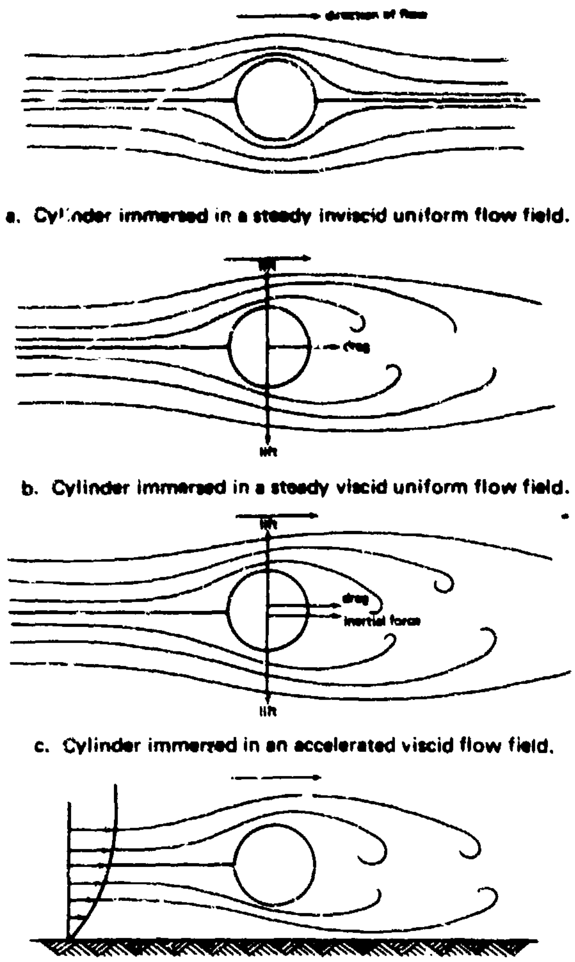
*At high Reynolds numbers, S = 0.27 should be considered as the dominant shedding frequency. Other investigators have noted higher frequency harmonics.



a. Cylinder immersed in a steady inviscid uniform flow field.

b. Cylinder immersed in a steady viscid uniform flow field.

c. Cylinder immersed in an accelerated viscid flow field.

d. Cylinder immersed in bottom boundary layer.

Figure 2. Types of flow about horizontal cylinders.

Upon contact with the bottom boundary, the flow underneath the cylinder stops, and the flow over the cylinder induces a positive (upward) lift force. Thus, a cylinder that is not restrained on a bottom boundary is unstable; that is, it can be alternately raised and lowered by the lift force due to flow asymmetry about the cylinder.

Forces on Cylinders in Oscillatory Flow

In unsteady oscillatory flow, such as that induced by waves, considerable complexity is added to the flow phenomena around horizontal cylinders. For example, the drag force, which is known to depend upon the N_R in steady flow has been shown by Keulegan and Carpenter (1958) for oscillatory flow to have no clear dependency on N_R for the range of this parameter in their tests. As will be discussed later, in their experiments on cylinders unaffected by wall effects, it was shown that the drag coefficient was dependent upon the dimensionless period parameter $U_m T/D$, where U_m is

the maximum water particle velocity in a wave cycle, T is the period of the oscillatory motion and D is the cylinder diameter. It was also shown that the drag coefficient C_D and the inertia coefficient C_I varied within one cycle of the oscillatory flow. The inertia coefficient varied dramatically with a cyclic period twice that of the incident wave period These findings for C_D and C_I for cylinders in oscillatory flow contrast sharply with the previously discussed flow behavior noted for steady and accelerated flow Force coefficient variability is very likely due to the alternate buildup and destruction of vortex fields on either side of the test cylinders.

If significant cyclic water particle motion in the vertical direction is present as would be the case, for example, if the cylinder is situated in deep water and near enough to the surface to be affected by wave-induced particle motions – then consideration will have to be given to vertical inertial and drag forces and a horizontal "lift" force induced by the vertical water particle velocity. However, no laboratory experiments are known to have been made which include measurements of both vertical and horizontal flow oscillation effects.

Difficulties in relating forces on cylinders in oscillating flow to some basic flow parameter such as the Reynolds number are greatly compounded when the cylinder is resting upon or is in near proximity to a plane boundary. As was true for the cases of steady and accelerating flow, lift due to the flow asymmetry is present, and the boundary will also have a pronounced effect on the inertial and drag forces. To date, however, no experimental studies have been conducted which examine these effects in detail. Existing test data consist of horizontal and vertical force measurements which, with an applicable wave theory, are used to generate force coefficients. Disagreement in the literature as to what are the appropriate wave force coefficients for use in design is understandable when one considers the multitude of variables affecting the flow around horizontal cylinders when the flow is oscillating.

In summary, then, no theory exists for accurately describing the flow about cylinders in oscillating flow, not even for cases where bottom boundary effects can be ignored. Furthermore, measurements of water particle kinematics, vortex formation about a cylinder, and effects of the bottom boundary layer on the flow around cylinders resting upon or near the seafloor have rarely been made.

Airy Wave Theory

Most investigations of wave forces on horizontal cylinders have used linear (Airy) wave theory to predict wave particle kinematics. This theory has many computational advantages over higher order, nonlinear theories.* LeMehaute et al. (1968) conducted a series of carefully controlled wave tank tests and compared linear and nonlinear wave theories with the test results. They concluded that the Airy theory is the most accurate theory near the bottom for predicting horizontal water particle velocities. Thus, a technically sound rationale was established for choosing linear theory in studies of wave forces on pipelines.

Linear wave theory is based on the following assumptions:

1. The fluid is homogeneous and incompressible, and the forces due to surface tension are negligible.

*Nonlinear theories which have been used in some studies include cnoidal, solitary wave, Stokes' higher order, and Dean's stream function theories. See the discussion in the Shore Protection Manual, Vol 1, pp 2-33 through 2-62.

2. The flow is irrotational.

3. The bottom is impermeable and horizontal.

4. The wave amplitude is small compared to the wave length and water depth.

5. The pressure is constant along the sea/air interface.

In the linear theory, it is assumed that the water surface profile is given by:

$$\eta = \frac{H}{2} \cos(k x - \omega t) \tag{4}$$

where $k = 2\pi/L$, and $\omega = 2\pi/T$. The waves are sinusoidal, having a constant amplitude and period.

Linear theory yields the following results for the horizontal and vertical water particle velocities:*

$$u = \frac{H\pi}{T} \left(\frac{\cosh \frac{2\pi y}{L}}{\sinh \frac{2\pi d}{L}} \right) \cos(k x - \omega t) \tag{5a}$$

and

$$v = \frac{H\pi}{T} \left(\frac{\sinh \frac{2\pi y}{L}}{\sinh \frac{2\pi d}{L}} \right) \sin(k x - \omega t) \tag{5b}$$

where the maximum particle velocities and accelerations are

$$u_{max} = \frac{H\pi}{T} \left(\frac{\cosh \frac{2\pi y}{L}}{\sinh \frac{2\pi d}{L}} \right) \tag{6}$$

and

$$v_{max} = \frac{H\pi}{T} \left(\frac{\sinh \frac{2\pi y}{L}}{\sinh \frac{2\pi d}{L}} \right) \tag{7}$$

Comparable equations for the maximum horizontal and vertical water particle accelerations are, respectively:

*Refer to Dean and Harleman (1966) for derivations of the wave particle kinematics equations.

$$\dot{u}_{max} = \frac{2 H \pi^2}{T^2} \left(\frac{\cosh \frac{2 \pi y}{L}}{\sinh \frac{2 \pi d}{L}} \right) \tag{8}$$

and

$$\left| \dot{v}_{max} \right| = \frac{2 H \pi^2}{T^2} \left(\frac{\sinh \frac{2 \pi y}{L}}{\sinh \frac{2 \pi d}{L}} \right) \tag{9}$$

For deep water (that is, $d/L > 0.5$) the equations for computing the maximum water particle velocities and accelerations take the following form:

$$u_{max} = v_{max} = \left(\frac{\pi H}{T} \right) e^{2 \pi y/L} \tag{10}$$

and

$$\dot{u}_{max} = \dot{v}_{max} = \left(\frac{2 \pi^2 H}{T^2} \right) e^{2 \pi y/L} \tag{11}$$

In equations 10 and 11, the origin of the vertical coordinate y is at the mean water surface, and the positive direction for y is upward.

Given H, T, L, and d, these equations are solved for the particle velocities and accelerations at any desired vertical coordinate y.

Some Cautionary Remarks

At this point it is worthwhile to discuss a few of the effects of real waves on pipelines in the ocean.

1. Waves in the ocean never have the regular period and amplitude assumed in linear wave theory. An estimate of the error introduced by assuming that the cylinder force coefficients derived from small scale, essentially sinusoidal wave tank tests, can be applied to actual pipelines in the ocean is not possible due to the lack of any force measurements at sea.*

2. The forces on pipelines from breaking waves have been only superficially examined in the laboratory, and no design procedure has been offered.

3. The scour and deposition of sediment around bottom-resting pipelines are processes which are known to occur in the ocean but as far as is known have never been studied in laboratory tests. It is doubtful whether these processes could be studied adequately on a small scale.

*Typically, for the range of variables in small-scale wave tank tests, inertia forces dominate the velocity dependent forces. In full-scale design problems at sea, it is possible for the opposite to be true.

4. Data obtained in small-scale tank tests cannot accurately account for the effects of pipeline roughness, including roughness changes due to marine growth. It is known that surface roughness can affect the drag force.

In light of the uncertainties associated with the above phenomena, the best that one can do at the present time is to proceed cautiously using conservative estimates of the wave environment and cylinder force coefficients. This is the approach taken in this report.

REVIEW OF PREVIOUS WORK

Experimental work from which design aids for submarine pipelines can be drawn is unfortunately limited to small-scale laboratory measurements of hydrodynamic forces on horizontal cylinders. Field tests with pipelines in the ocean have not been conducted, but such tests are planned by researchers at the University of Hawaii and Oregon State University. Laboratory work which has resulted in data useful to the designer is reviewed in this section. Publications based on this laboratory work and other contributions to the determination of wave forces on pipelines are given in the annotated bibliography (Appendix). A complete bibliography is given in the Reference and Bibliography sections of this report.

With very few exceptions laboratory measurements of *horizontal wave forces* on horizontal cylinders have been made to determine force coefficients for use in the "Morison equation." This equation considers the total horizontal force as the sum of the horizontal drag force and horizontal inertial force. The drag force is a function of the drag coefficient C_D and the horizontal water particle velocity u, as shown in Equation 1. The inertial force is a function of the inertial coefficient C_I and the horizontal water particle acceleration $\dot{u}$, as shown in Equation 3. Thus, the horizontal force per unit length of pipe is:

$$F_H = \frac{C_D}{2} \rho D u^2 + \frac{C_I \rho \pi D^2 \dot{u}}{4} \tag{12}$$

where ρ = mass density of water

D = diameter of the pipe

There is much less agreement among researchers on the analysis of *vertical wave force* data from laboratory tests with horizontal cylinders; most accept that a lift force, which is perpendicular to the direction of water particle velocity, exists in some cases. This lift force is a function of a lift coefficient C_L and the square of the water particle velocity. Some have analyzed vertical wave force as including a vertical inertial component, a function of a vertical inertial coefficient C_W.

Wiegel (1965) suggests a Morison-type formulation for vertical wave force. The same coefficients, C_D and C_I, used for horizontal forces are applied to vertical velocity v and acceleration $\dot{v}$. The problem with this approach is that it cannot be applied to pipelines on the seafloor because there the vertical velocity and acceleration are theoretically zero.

Some researchers have chosen to consider horizontal and vertical wave forces each as functions of only one term. These functions often involve a horizontal force coefficient C_H or a vertical force coefficient C_V.

For the purposes of this report, the previous work is categorized based on the proximity of the cylinder (pipeline) to a parallel solid boundary (seafloor). The first category is the cylinder remote from the solid boundary; that is, the seafloor or the bottom of the test tank has no effect on the flow around the pipeline. The second category is the cylinder in contact with the solid boundary; that is, the pipeline is resting on the seafloor or the floor of the test tank. The third category is the cylinder near the boundary; that is, the pipeline is supported above the seafloor or the floor of the test tank close enough to have the flow around it affected by the boundary. The following review of previous work addresses the situation of pipeline orientation parallel to the wave front, unless it is noted otherwise. This parallel orientation represents the most severe wave force situation, so it is of primary concern to the designer.

Cylinders Remote From Boundary

A major step toward a means of predicting the wave forces on a horizontal cylinder remote from boundaries was taken by Keulegan and Carpenter (1958). They first introduced the period parameter, $U_m T/D$, as a dimensionless parameter against which Morison force coefficients could be plotted, and experimentally determined C_D and C_I for period parameters below 125. Here, U_m is the amplitude of the oscillating fluid particle velocity, T is the period of the oscillation, and D is the diameter of the cylinder. Tests performed by Sarpkaya (1975) for period parameters below 50 closely corroborated Keulegan and Carpenter's results for drag coefficients. Sarpkaya's results for inertial coefficients were slightly higher than Keulegan and Carpenter's in the range of period parameters from 12 to 27, and lower beyond 27. None of these authors found any correlation with Reynolds number. The higher of these results show $C_D = 2.3$ at $U_m T/D = 12$, $C_D = 1.4$ at 50, and approaching an asymptote of $C_D = 1.2$ beyond $U_m T/D = 125$. C_I values taken from Keulegan and Carpenter and Sarpkaya, whichever value is higher, are a maximum of $C_I = 2.2$ at $U_m T/D = 2$, a minimum of $C_I = 0.9$ at 12, increasing rapidly to 1.3 at 18, and then gradually to an asymptote at a value of C_I over 2.5 beyond $U_m T/D = 125$. Keulegan and Carpenter based their curve for the higher values of $U_m T/D$ on very few data points. Sarpkaya's data for C_I seems to approach an asymptote of 1.3. Keulegan and Carpenter did not determine lift coefficients but Sarpkaya did for period parameters below 50. He found peaks of about $C_L = 2.$[illegible] at $U_m T/D$ of 10 and 18, decreasing gradually to about $C_L = 1.0$ at $U_m T/D$ of 50.

Rance (1969b) contends that wave forces are related to Reynolds number. He maintains that the period parameter relationship is far less important for higher Reynolds numbers. He determined that for Reynolds numbers greater than 6×10^5 the lift coefficient, as he defined it, is less than 0.2. This value of lift coefficient when converted to C_L as defined here equals 1.

The characteristics of these tests and others are shown in Table 1.

Cylinders in Contact With Boundary

Recent (since 1970) experiments on wave forces on horizontal cylinders in contact with a lower boundary have provided very useful information for submarine pipeline designers. Researchers from the University of Hawaii, the Naval Postgraduate School, and Oregon State University have contributed.

Table 1. Test Conditions by Various Investigators for Cylinders Remote From Boundary

Investigators	Cylinder Diameter (in.)	Wave Height (in.)	Wavelength (in.)	Period (sec)	Water Depth of Pipe (in.)	Clearance of Pipe Above Bottom (in.)	Reynolds Number $\times 10^4$
Keulegan & Carpenter (1958)	0.5 to 3.0	2.3 to 16.9	[a]	2.075	9.8	17.7	0.4 to 2.9
Rance (1969)	1 to 12	[b]	–	4 to 14	45	45	8.0 to 60.0
Al-Kazily (1972)	1.5 and 4.0	0.9 to 2.6	–	0.47 to 1.33	3.5 to 22.8	–	<1.5
Brater & Wallace (1972)	1.25 to 2.38	1.32 to 3.72	31 to 73	–	3 to 9	3 to 9	0.1 to 2.0
Sarpkaya (1975)	1 to 2.5	[c]	–	2.86	9 to 10	9 to 10	0.3 to 5.0

[a]Tests in standing wave tank.

[b]Tests in pulsating water tunnel.

[c]Tests in vertical water tunnel.

Table 2. Test Conditions by Various Investigators for Cylinders in Contact With Boundary[a]

Investigators	Cylinder Diameter (in.)	Wave Height (in.)	Wavelength (in.)	Period (sec)	Water Depth (in.)	Clearance of Pipe Above Bottom (in.)	Reynolds Number x 10^4
Johansson & Reinius (1963)	2	6.0 to 9.2	–	3.0	13.8 to 21.7	0	–
Johnson (1970)	4.5	0.99 to 9.27	–	–	5 to 14	"nearly touched"	–
Grace (1971)	3	7.3 to 13.9	–	2.05 to 4.15	36	1/32 and 1/8	1.9 to 3.7
Schiller (1971)	6	0.21 to 5.32	23 to 209	0.56 to 2.70	12, 15, and 18	1/8	–
Yamamoto et al. (1973)	2, 4.5, and 6	0.88 to 3.7	16 to 103	0.60 to 1.5	10 and 14.125	1/8	0.2 to 3.0

[a]Cylinders having 1/8 in. or less clearance considered in contact with boundary.

Grace (1971a) performed experiments with a horizontal pipe on the floor of a wave tank and derived maximum force coefficients of $C_I = 3.5$ and $C_D = 2.5$. Grace utilized data from Priest (1961) for bottom-laid pipes and derived mean maximum force coefficients of $C_I = 4.7$, $C_D = 2.4$ and $C_L = 1.5$. Grace also processed data from Johansson (1968) and found force coefficients for a pipe on the bottom of $C_I = 3.3$, $C_D = 2.0$, and $C_L = 4.0$.

Yamamoto, Nath, and Slotta (1973a) performed experiments with horizontal cylinders on the bottom of a wave tank. They analyzed their data and that of Schiller (1971). From their data they found maximum force coefficients of $C_I = 5.95$ and $C_L = -8.43$ (downward-acting lift force). These authors found that horizontal and vertical inertial coefficients are equal for a cylinder near the boundary. They also defined three regions of horizontal forces on bottom-resting cylinders based on the relative magnitudes of the inertia and the drag components. They did this on a plot of H/D versus h/L.

The characteristics of these tests and others are shown in Table 2.

Cylinders Near Boundary

Experiments on horizontal cylinders away from the bottom boundary less than two pipeline diameters (that is, $e/D < 2$) have been performed. Such testing has usually been done in conjunction with tests on pipelines on the seafloor, which were discussed earlier.

Grace (1971a) performed tests of this type for e/D values below 0.417 and found C_D values between 2 and 3.6 and C_I values from 3.54 at $e/D = 0.083$ to 1.17 at $e/D = 0.292$. There was no apparent relationship between C_D and e/D, but C_I seemed to decrease with increasing e/D. Grace also processed data from Johansson (1968) for e/D values of 0.2[illegible] and 1.00 and found C_I values equal to 2.4 and 2.0, respectively. The value of C_D was found to equal 1.1 for both values of e/D. Experiments by Schiller (1971) and Yamamoto, et al. (1973a), with e/D approximately 0.5, gave C_I values between 2.1 and 2.7 and $C_L = 0$.

The characteristics of these and other tests are given in Table 3.

Table 3. Test Conditions for Various Investigators for Cylinders Near Boundary

Investigators	Cylinder Diameter (in.)	Wave Height (in.)	Wavelength (in.)	Period (sec)	Water Depth (in.)	Clearance (in.)	e/D
Grace (1971)	3	7.3 to 13.9	–	2.05 to 4.15	36	1/4 to 1-1/4	0.083 to 0.417
Schiller (1971)	6	0.65 to 5.32	19 to 190	0.56 to 2.40	18	3	0.5
Muellenhoff & Slotta (1971)	5.5	0.5 to 3.5	10 to 80	0.5 to 1.5	12	–	–
Yamamoto et al. (1973)	6.0	1.5 to 3.7	44 to 89	0.93 to 1.30	14-1/8	3-1/8	0.52

Orientation Effects

A few authors have experimentally assessed the effects on wave forces of the orientation of the pipeline relative to the incident waves. Johansson and Reinius (1963) and Grace (1971a) found that both the horizontal and vertical wave forces are maximum when the pipeline is parallel to the wave front. Al-Kazily (1972) found a nonperiodic uplift force on cylinders perpendicular to the wave front. Denson and Priest (1974) found a 10% increase in maximum drag force at a cylinder orientation of 70 degrees relative to wave fronts. They found that lift forces decreased rapidly for orientations less than 90 degrees, but they did not test for the 0-degree case (cylinder perpendicular to wave front).

Previous data are limited, but present indications are that both horizontal and vertical forces can be considered maximum for cylinders oriented parallel to the wave fronts.

DESIGN METHODS

Procedure

Until a clear-cut theory is developed to fully describe wave forces on pipelines (per unit length of pipe), the following equations taken from the above discussion and a review of the literature are offered:

$$F_H = F_{HD} + F_{HI} + F_{HLA} + F_{HLE} \qquad (13)^*$$

$$F_V = F_{VD} + F_{VI} + F_{VLA} + F_{VLE} \qquad (14)$$

where

$$F_{HD} = \frac{C_D \rho D u^2}{2} \qquad (15)$$

$$F_{HI} = \frac{C_I \rho \pi D^2 \dot{u}}{4} \qquad (16)$$

$$F_{HLA} = \frac{C_{LA} \rho D v^2}{2} \qquad (17)$$

$$F_{HLE} = \frac{C_{LE} \rho D v^2}{2} \qquad (18)$$

$$F_{VD} = \frac{C_D \rho D v^2}{2} \qquad (19)$$

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* Lift in F_{HLA} and F_{HLE} is defined as a force which acts on a cylinder in a direction normal to the water particle velocity.

$$F_{VI} = \frac{C_I \rho \pi D^2 \dot{v}}{4} \tag{20}$$

$$F_{VLA} = \frac{C_{LA} \rho D u^2}{2} \tag{21}$$

$$F_{VLE} = \frac{C_{LE} \rho D u^2}{2} \tag{22}$$

where D = drag

I = inertia

LA = lift due to flow asymmetry

LE = lift due to vortex shedding

Equations 13 and 14 are solved by using the appropriate wave force coefficients from Design Aids I, II, and III, which will be introduced shortly. The water particle kinematics (that is, u, v, $\dot{u}$, and $\dot{v}$) are computed by Airy wave theory.

For shallow and intermediate depth water, $d/L < 0.5$, the Airy wave theory equations for shallow water wave particle velocity and acceleration (Equations 6 through 9) should be used; while the deep water form of these equations (Equations 10 and 11) may be used in situations where $d/L > 0.5$.

It should be remembered that a phase difference exists between maximum values of the force terms in Equations 13 and 14. Neglecting the lift force terms for the moment, the horizontal and vertical force equations can be written as follows (See Wiegel, 1964, pp 254-256):

$$F_H = -2\pi^2 \rho C_I \left(\frac{\pi D^2}{4}\right) \frac{H}{T^2} \left(\frac{\cosh \frac{2\pi y}{L}}{\sinh \frac{2\pi d}{L}}\right) \sin \omega t$$

$$+ \frac{1}{2} \rho C_D D \left(\frac{\pi^2 H^2}{T^2}\right) \left(\frac{\cosh \frac{2\pi y}{L}}{\sinh \frac{2\pi d}{L}}\right)^2 \left|\cos \omega t\right| \cos \omega t \tag{23}$$

$$F_V = -2\pi^2 \rho C_I \left(\frac{\pi D^2}{4}\right) \frac{H}{T^2} \left(\frac{\sinh \frac{2\pi y}{L}}{\sinh \frac{2\pi d}{L}}\right) \cos \omega t$$

$$- \frac{1}{2} \rho C_D D \left(\frac{\pi^2 H^2}{T^2}\right) \left(\frac{\sinh \frac{2\pi y}{L}}{\sinh \frac{2\pi d}{L}}\right)^2 \left|\sin \omega t\right| \sin \omega t \tag{24}$$

The phase angle at which the maximum horizontal force occurs can be found by differentiating Equation 23 with respect to ωt and setting the result equal to zero. Thus,

$$\sin \beta_{HM} = \pm \frac{\pi D}{2H}\left(\frac{C_I}{C_D}\right)\frac{\sinh \frac{2\pi d}{L}}{\cosh \frac{2\pi y}{L}}$$

where $\cos \beta_{HM} = 0$ if

$$\frac{\pi D}{2H}\left(\frac{C_I}{C_D}\right)\frac{\sinh \frac{2\pi d}{L}}{\cosh \frac{2\pi y}{L}} > 1$$

In cases where $\cos \beta_{HM} = 0$, the maximum force is entirely inertial.

In a similar fashion, for the vertical inertial and drag force terms, the critical phase angle is:

$$\cos \beta_{VM} = \pm \frac{\pi D}{2H}\left(\frac{C_I}{C_D}\right)\frac{\sinh \frac{2\pi d}{L}}{\sinh \frac{2\pi y}{L}}$$

where if

$$\frac{\pi D}{2H}\left(\frac{C_I}{C_D}\right)\frac{\sinh \frac{2\pi d}{L}}{\sinh \frac{2\pi y}{L}} > 1$$

$$\sin \beta_{VM} = 0$$

The question of what critical phase angle to use in cases where lift forces are significant has not been resolved. One design approach (conservative) would be to simply compute the maximum lift force independent of the computation for combined drag and inertial force and add the two results.

In general, two types of lift force must be accounted for. The first, due to flow asymmetry F_{LA}, acts upward on pipelines in contact with the seafloor and downward for pipelines having a gap-to-diameter ratio $e/D < 2.0$. This lift force component is negligible for cases where $e/D > 2.0$.* The second type of lift force F_{LE} is caused by vortex shedding. In some design situations this force component can have an appreciable amplitude; its dominant frequency of oscillation, however, is high compared

*Based on the results of potential flow theory for uniformly accelerated cylinders near a boundary.

to that of the incident wave frequency and can be estimated from the Strouhal number, $S = n\,D/V$, where $S = 0.27$ for Reynolds number flows greater than 5×10^5.*

Aids

The remainder of this section of the report will be devoted to discussing three design aids which can be used in selecting appropriate wave force coefficients.

Experiments in waves and oscillatory flow on horizontal cylinders and model pipelines have resulted in force coefficients of drag, inertia, and lift. Unfortunately, these test results cannot be applied directly to the design of prototype pipelines, because the range of laboratory test parameters frequently does not match the foreseeable range of submarine pipeline parameters. The test results have been used in Design Aids I, II, and III and have been used to guide the selection of coefficients beyond the test range.

Design Aid I. Design Aid I (Figure 3) is for cylinders remote from the boundary; i.e., where $e/D > 2$. Test results from Keulegan and Carpenter (1958) and Sarpkaya (1975) were adapted for use in the preparation of this design aid. This design aid is recommended for use with period parameters less than 125. For period parameters in excess of 125, a drag coefficient of 1.2 (that for subcritical steady flow) seems reasonable. This drag coefficient is appropriate because it is hypothesized that the drag coefficient in this range exhibits a similar trend to that of steady flow. The well-known steady flow relationship of drag coefficient with Reynolds number shows a significant drop in drag coefficient beyond a Reynolds number of 10^5. Until this drop is verified for horizontal cylinders in oscillatory flow at higher Reynolds numbers, it is recommended that a conservative value of $C_D = 1.2$ be used.

Inertia coefficients (C_I) for cylinders remote from the boundary are also given in Design Aid I, for period parameters less than 125. This data is also the result of work by Keulegan and Carpenter, except at the higher period parameters where there are fewer data points to support their curve. Sarpkaya has confirmed Keulegan and Carpenter's data at period parameters less than 12. In the range of period parameters from about 12 to 25 Sarpkaya shows slightly higher values of the inertia coefficient, and for period parameters between 25 and 50 Sarpkaya shows significantly lower values of inertia coefficient. The higher values of C_I from these authors are recommended in Design Aid I except at period parameters beyond approximately 65 where Keulegan and Carpenter's curve crosses $C_I = 2.0$. It is recommended that for higher period parameters a value of C_I of 2.0 be used in design problems based on potential flow theory as discussed for cylinders near the boundary.

Keulegan and Carpenter did not study lift coefficients; however, Sarpkaya did. He gave his results as shown in Design Aid I for period parameters less than 50. At $U_m T/D = 50$, C_L is about 1. From the Rance (1969b) data it was determined that C_L is less than 1 for Reynolds number beyond 6×10^5. It is, therefore, recommended for design that $C_L = 1$ be used for period parameters greater than 50.

*Strictly speaking, this expression is only applicable to uniform flows. Vortex shedding frequencies computed for oscillatory flow should be treated as rough estimates.

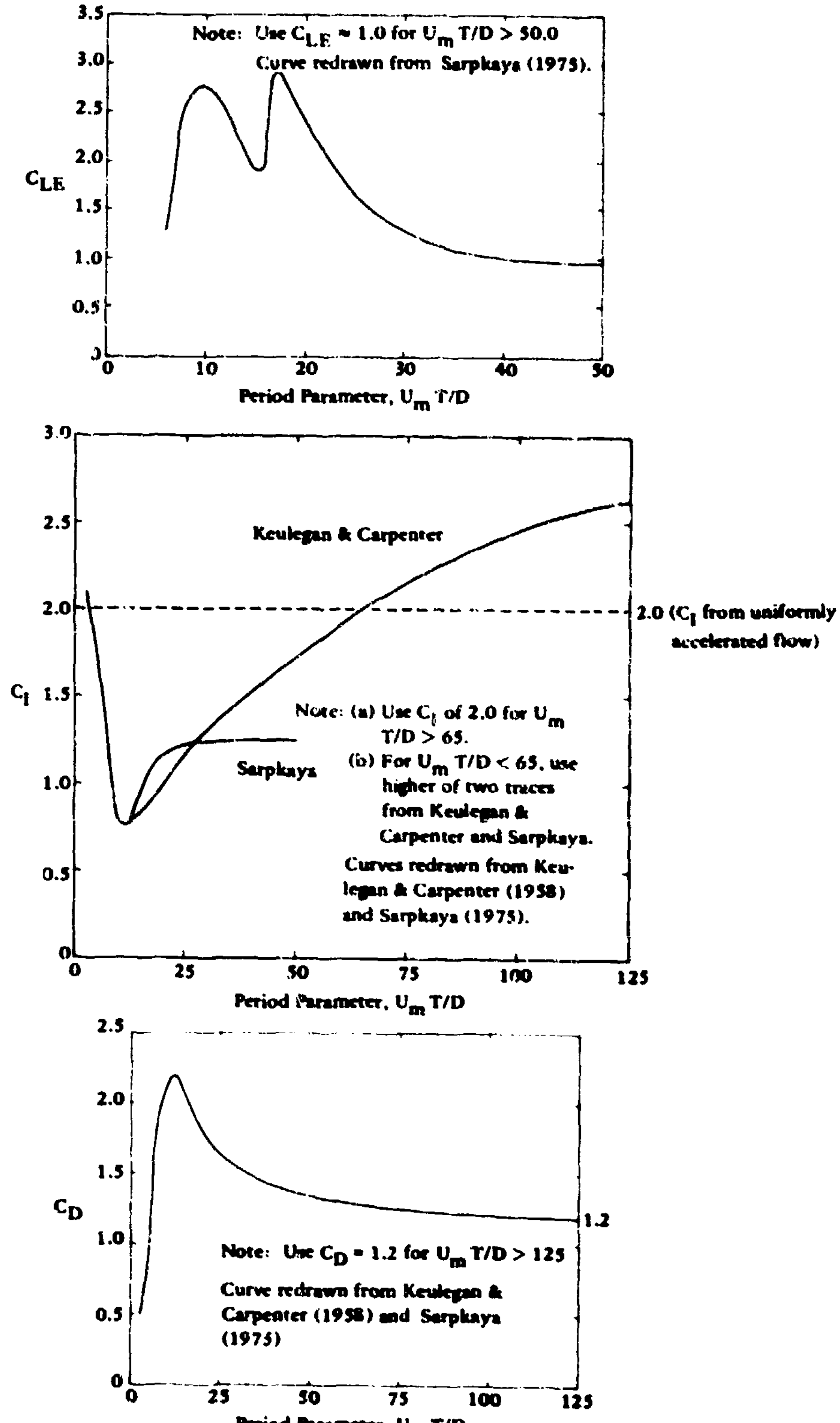


Figure 3. Design Aid I – force coefficients for pipe remote from bottom.

Design Aid II. Design Aid II (Figure 4), which provides force coefficients for cylinders resting on the bottom, was adapted from Figure 24 of Yamamoto et al. (1973a). The chart is divided into three regions, depending upon the relative importance of acceleration- and velocity-dependent forces. Region A in the upper left-hand corner of the plot applies to conditions where the velocity dependent forces (that is, drag and lift) dominate over the inertial forces. In this region the inertia force amounts to less than 5% of the drag force. In general, high waves in shallow water create the conditions applicable to this region. In the lower right-hand corner, Region C, the situation is reversed; the inertial forces dominate over the velocity dependent forces. In this region, the drag force amounts to less than 5% of the inertia force. Between these two regions, a band (Region B) exists, wherein both inertial and velocity dependent forces should be considered in the design problem.

The choice of force coefficients appearing on Design Aid II was based on the experimental work by Grace, Schiller, and Yamamoto et al., which was reviewed earlier. Also considered were force coefficient suggestions by Grace (1973) and Yamamoto et al. (1973a). Average force coefficients of $C_D = 1.5$ and $C_I = 3.9$ were used in the construction of this design aid.

Little is known about the range of drag coefficient C_D for cylinders in contact with the bottom in oscillatory flow. Although Grace (1971a) found values of drag coefficient as high as 2.5, he cautioned that these values were determined from tests at Reynolds numbers much lower than those in the prototype pipeline situation. Well-documented experimental results for uniform flow (far removed from a plane boundary) give $C_D = 1.2$. It is probably conservative to assume this as a minimum value for pipes on the bottom for high Reynolds number flows. Therefore, a range of C_D from 1.0 to 2.0, as Grace (1973) suggests, is recommended in Design Aid II where drag is significant (Regions A and B).

Yamamoto et al. (1974b) showed that potential flow theory predicts an inertia coefficient C_I of 3.29 for a cylinder resting on the bottom in accelerated flow. This value agrees with the minimum from the previous laboratory work reviewed earlier which found values in the range of 3.3 to 4.7 and one value in excess of 5.5. Grace (1973) suggested maximum values of $C_I = 5.0$ and $C_W = 6.0$, but the justification for these very high values and the need for using different horizontal and vertical inertia coefficients are not clear. It is recommended in Design Aid II that a range of inertia coefficient of 3.3 to 4.5 be used where inertia forces are significant (Regions B and C).

Most previous laboratory investigations yielded lift coefficients less than the $C_L = 4.49$ predicted for potential flow for a cylinder in contact with a plane boundary in uniform flow. Grace (1973) suggested a maximum $C_L = 2.0$. It is recommended that the approximate potential flow value of $C_L = 4.5$ be used for design as shown in Design Aid II.

Design Aid III. Design Aid III (Figure 5) is for cylinders near the boundary. If a cylinder is removed more than two cylinder diameters from a plane boundary, then the force coefficients recommended earlier can be used for design purposes. However, if the clearance-to-diameter ratio lies in the range $0 < e/D \leq 2.0$, then adjustments will have to be made to the force coefficients due to the effects of flow asymmetry about the cylinder.

The data on drag coefficients for low e/D ratios is so widely scattered that little can be deduced from them for application to design. Furthermore, a small gap (e) under a pipeline near a typical seafloor can often be widened by scouring or closed by

deposition. Therefore, the recommended range of design values for C_D for a cylinder near the bottom, a shown in Design Aid III, provide for the full range of $0 \leq e/D \leq 2$. It is recommended that a C_D value between 1.2 (remote from bottom) and 2.0 (the maximum for contact with bottom) be used for design.

The experimental results of Yamamoto et al. (1973) and Schiller (1971) – though somewhat limited in the range of e/D values investigated – tend to support the results obtained from potential flow about cylinders near a boundary. Until additional experimental data becomes available, it is recommended – with certain reservations stated below – that the designer use the C_I and C_{LA} values presented in Design Aid III.

Design Aid III for C_I is based on potential flow solutions. The curve for C_I decreases from 3.3 (to 4.5) for e/D = 0 (cylinder in contact with bottom) to 2.0 for e/D = 2 (cylinder remote from bottom).

It should be recalled from the discussion of force coefficients on cylinders remote from a boundary that a lift force occurs which is attributable to vortex shedding. Until experimental data are available which prove otherwise, the same should be assumed for cylinders near a boundary. The choice of lift coefficient due to vortex shedding should be made by the procedure outlined in the discussion of cylinders remote from the boundary.

Note that small gap-to-diameter ratios (0.1 or less) lead to high negative values for C_{LA}. The designer is cautioned, however, against using large negative C_{LA} values. Scour or deposition could easily widen or close the gap. As would be expected, the curve in Design Aid III for C_L asymptotically approaches a value of 0 as e/D increases.

FINDINGS

1. For the prediction of wave forces on submarine pipelines, the Morison equation for horizontal forces and the lift equation for vertical forces have been broadly accepted as reasonable for design.

2. It is generally agreed that the expressions for particle velocity and acceleration derived from the Airy wave theory, despite their limitations, are useful for the prediction of wave kinematics for use in wave force estimates. Most researchers have used this theory to analyze wave force data and to derive wave force coefficients.

3. The use of force equations requires empirical estimates of wave force coefficients. However, very little data is available for wave forces on submerged pipelines from which these coefficients can be determined. All of the pertinent data were obtained during small-scale tests conducted in the laboratory. There are disagreements and inconsistencies in some of the existing data which are attributable to poor test techniques, questionable data analysis, or the limited range of test variables.

4. Potential flow theory for uniform and accelerated flow past horizontal cylinders has been used with some success to reveal trends from which lift and inertial wave force coefficients for pipelines can be estimated.

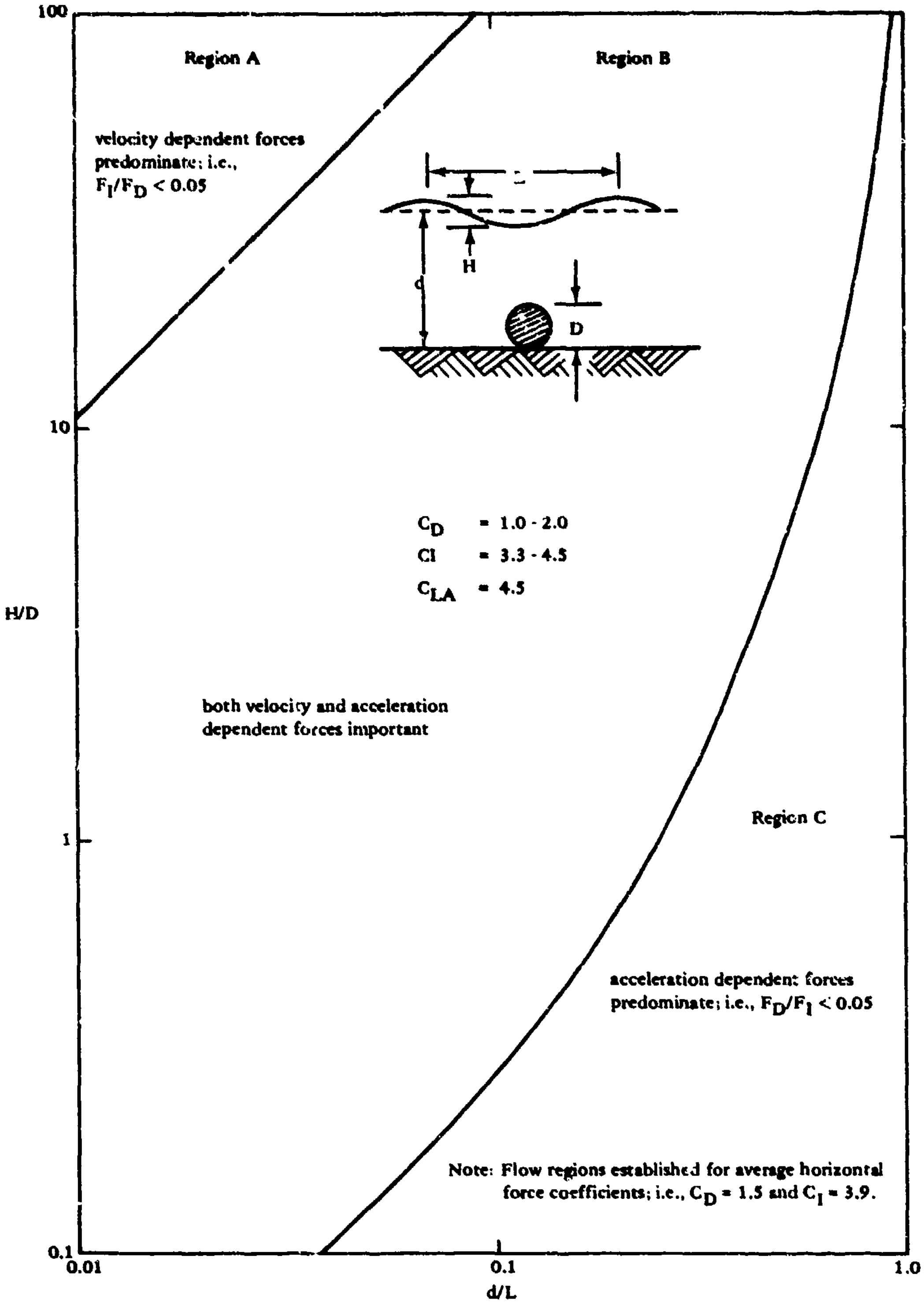
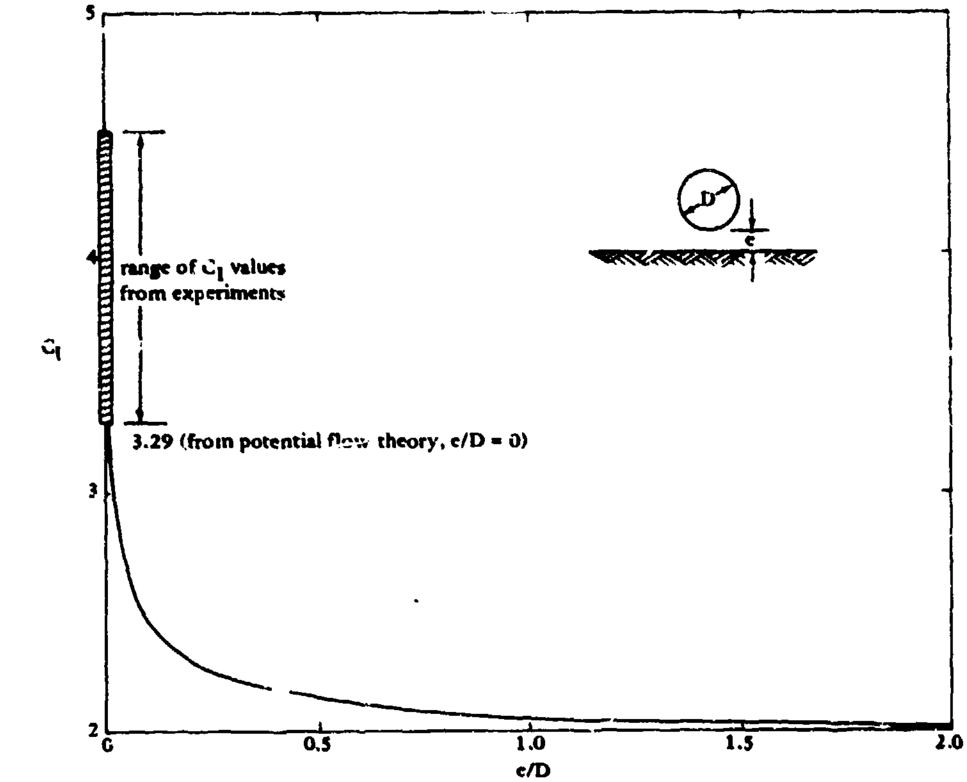


Figure 4. Design Aid II -- force coefficients for pipe on bottom.



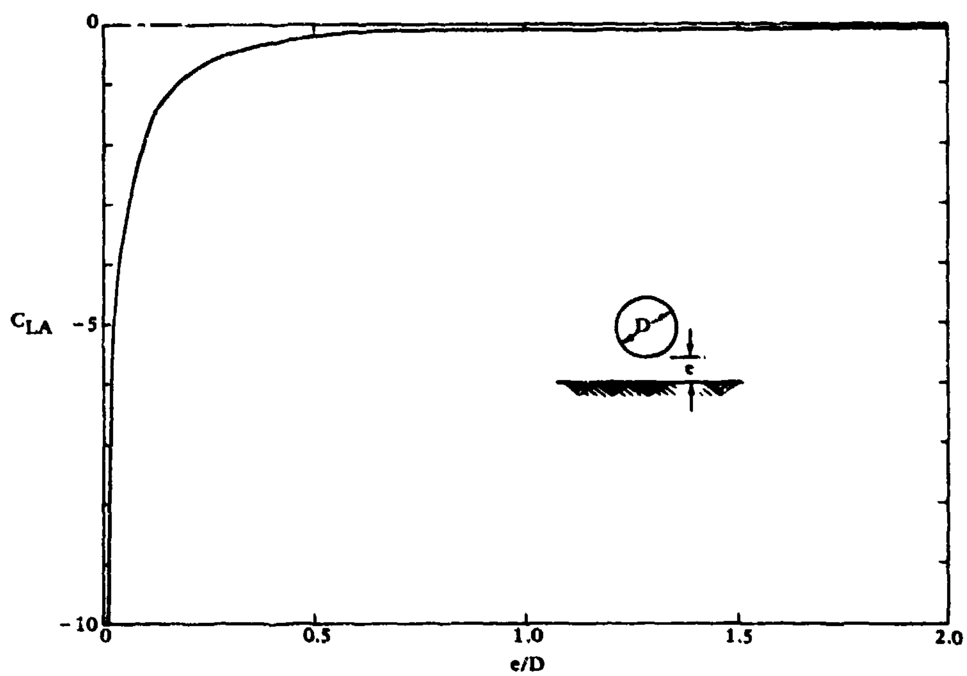


Figure 5. Design Aid III – force coefficients for pipe near bottom.

RECOMMENDATIONS

Until additional tests and analyses are performed as outlined below, it is recommended that (1) Equations 13 and 14, (2) the force coefficients given in Design Aids I, II, and III, and (3) the Airy wave theory (water particle kinematics) be used as an interim design procedure.

Additional tests and analyses are needed to provide greater confidence in the force coefficients used in the design of submarine pipelines. Needed tests are listed below in descending order of priority:

1. Larger scale model tests and prototype tests at sea (needed to provide estimates of wave force coefficients)

   (a) For period parameters $U_m T/D$ in excess of 125 for inertia and drag and period parameters greater than 50 for lift (cylinders remote from a boundary).

   (b) For Reynolds numbers $U_m D/\nu$ greater than 10^5 (all three classes of cylinder problems).

   (c) For cylinder gap-to-diameter ratios $e/D < 2$.

2. Breaking wave tests, since there is virtually no data on breaking wave forces on pipelines

3. Tests to determine scour and deposition of seafloor materials around submarine pipelines since such phenomena can have a pronounced effect on wave induced forces

4. Tests to determine the effects of pipeline roughness on wave-induced forces

5. Tests to determine lift forces on pipelines perpendicular to wave fronts, because little is known about these forces for this orientation of nearshore submarine pipelines.

It is also recommended that the interim design procedures recommended in this report be updated based on the results of future tests and analyses if changes are warranted.

Appendix

ANNOTATED BIBLIOGRAPHY

The symbols used by the original authors have been preserved in these annotations although they sometimes differ from those used in the text of this report.

Al-Kazily Fadhil, M. (1972). Forces on Submerged pipelines induced by water waves, University of California, Hydraulic Engineering Laboratory, Technical Report HEL 9-21. Berkeley, CA, Oct 1972.

The purpose of this study was to produce design data for buoyant, anchored pipelines subjected to wave-induced forces. The pipelines are considered to be supported at a sufficient height above the seafloor as to be unaffected by cylinder/seafloor interaction with wave particle kinematics.

A series of wave tank tests produced force data for horizontal cylinders aligned with axes parallel and perpendicular to the wave crests. Data was also obtained for an inclined cylinder lying in a vertical plane at a right angle to the incident waves.

The author uses the Morison equation and linear wave theory to develop analytical expressions for estimating cylinder mass and drag coefficients (C_M and C_D, respectively) as a function of wave-induced forces. The author considers his computed C_D values to be unrealistically large; maximum C_M values obtained are around 2.0 to 2.5, depending on test conditions.

In agreement with an earlier study conducted by Keulegan and Carpenter (1958), the author finds that both C_M and C_D vary within a wave cycle. Cylinder coefficients were also found to vary with the wave height, wave period, cylinder diameter, and depth of cylinder submergence.

It was concluded that·

1. When the cylinder is at a right angle to the wave crests, it experiences a nonperiodic uplift force. The uplift force becomes the dominant force acting on cylinders longer than half a wavelength.

2. The maximum force acting on an inclined circular cylinder lying in a vertical plane at a right angle to the wave crests is independent of the angle of inclination.

This report includes a review of previous work, an extended discussion of the methodology used to obtain added mass and damping coefficients and a complete compilation of recorded data.

Beckmann, H., and M. H. Thibodeaux (1962). "Wave force coefficients for offshore pipelines," in Proceedings of ASCE, Journal of Waterways and Harbors Division, vol 88, no. WW2, May 1962, pp 125-138.

The authors derived drag, lift, and inertia force equations for wave forces on pipelines in contact with smooth hard-surfaced ocean floors. It was shown that two hydrodynamic phenomena are present in oscillating flow around bodies attached to flat walls (waves on pipelines on the seafloor): (1) the presence of the wall causes a reduction in the drag coefficient, and (2) only fully developed turbulent flow configurations are permitted. It was also shown that for the case presented here inertial forces are not dominant.

The wave force equations derived by the authors were:

$$F(\text{drag}) = h C_D \frac{\rho u^2}{2}$$

$$F(\text{lift}) = h C_L \frac{\rho u^2}{2} f$$

$$F(\text{inertia}) = C_M \frac{\pi h^2 \rho}{4} \left(\frac{\partial u}{\partial t}\right)$$

where h = height of pipe

ρ = density of water

u = velocity component normal to pipe

f = Coulomb friction factor

The force coefficients which appear in these equations, C_D, C_L, and C_M, were determined from the literature.

The recommended values of the force coefficients for rough pipelines on the seafloor were

$$C_D = 0.5 \qquad C_L = 0.5 \qquad C_M = 1.0 \text{ to } 2.0$$

A combined drag and lift force coefficient,

$$C = \sqrt{C_L^2 + C_D^2}$$

was shown to be equal to 0.7.

For further discussion of this article, see Wilson and Reed (1963) in this Appendix.

Wilson, B. W., and R. O. Reid (1963). "Discussion of 'Wave force coefficients for offshore pipelines,' by H. Beckmann and M. H. Thibodeaux," in Proceedings of ASCE, Journal of Waterways and Harbors Division, vol 89, no. WW1, Feb 1963, pp 61-65.

The authors contend that the wave force coefficients recommended by Beckmann and Thibodeaux are too low, and more conservative values of these coefficients should be used for design. To support this contention the authors offer (1) a table of drag and inertia coefficients from the experiments of other researchers on accelerating flow past circular cylinders, and (2) potential flow-theory-derived coefficients of inertia and lift.

The table gives drag coefficients C_D between 0.4 and 1.6. In the ocean, pipelines tend to accumulate coatings of fauna and flora and, thus, lose their initial smoothness; as a result C_D increases. Therefore, the authors recommend C_D values at least ≥ 1.0.

The authors showed from potential flow theory and the basic definition of the coefficient of lift C_L that $0.74 < C_L \leq 4.48$. They suggested from this result and previous work that a value of $C_L \geq 1.0$ be adopted for design.

The authors point out that from potential flow theory the inertia coefficient C_M is 2.00 for a cylinder away from a wall and 3.30 for one in contact with a wall. From the table the average C_M of cylinders remote from a wall is 1.5. By applying the ratio 3.30:2.00 to this average, a C_M value of 2.5 was estimated.

For a rebuttal to this discussion, see Beckmann and Thibodeaux (1963) in this Appendix.

Beckmann, H., and M. H. Thibodeaux (1963). "Closure of wave force coefficients for offshore pipelines," in Proceedings of ASCE, Journal of Waterways and Harbors Division, vol 89, no. WW3, Aug 1963, pp 53-55.

The authors deny Wilson and Reid's contention that the force coefficients recommended in the original paper were too low. They maintain that Wilson's table of experimentally determined drag coefficients C_D and inertia coefficient C_M displayed much scatter, suggesting that these data may not be valid. They concur that flora and fauna will settle on the pipeline and would increase C_D if the seafloor were not similarly roughened. However, flora and fauna will also settle on the seafloor, causing low energy boundary layer flows that result in C_D which is lower, not higher, as suggested by Wilson and Reid.

The authors question the use of the potential flow theory in estimating the coefficients of lift C_L and of mass C_M. They maintain that the actual flow conditions around offshore pipelines are quite different from the assumed potential flow particularly at the "stagnation points." It is their opinion that C_L is controlled by the pressures, not the velocity distributions, and that C_M is the same for a free cylinder as it is for a cylinder attached to a wall.

Brater, E. F., and R. Wallace (1972). "Wave forces on submerged pipelines," in Proceedings of the Thirteenth Coastal Engineering Conference, Vancouver, BC, 1972, Chapter 95, pp 1,703-1,722.

The authors performed wave tank experiments with models of submerged pipes. Continuous records of horizontal forces on four 1/75-scale model pipes, ranging in diameter from 0.104 to 0.198 foot, were measured in waves 0.11 to 0.31 foot high and 2.6 to 6.1 feet long. The pipes were placed in the 1-foot-deep tank 0.25 to 0.75 foot above the bottom, on the bottom, and below the bottom (half a pipe diameter to 0.2 foot). These 1/75-scale model tests were intended to simulate pipe 8 to 15 feet in diameter in waves 8 to 23 feet high with periods from 6 to 12 seconds.

Inertial and drag coefficients for use in the Morison equation to estimate horizontal wave forces on pipelines were determined using Airy Theory for water particle velocities and accelerations. By assuming that drag forces were negligible at the phase angles of maximum horizontal force (which occurred between 65 and 77 degrees), the authors calculated maximum values of the inertial coefficient for each test condition. By assuming that the horizontal forces at phase angles of 0 and 180 degrees, were primarily the result of drag, the authors calculated values of drag coefficient for each test condition.

The inertial coefficients were found to vary in an orderly linear manner with $-Z/L$ (where $-Z$ = depth of the center of the pipe under the water surface and L = wave length), indicating that the proximity of the water surface is significant in these tests. Optimized least square fits for these data were determined, and linear equations were derived. The authors considered these inertial coefficients to be the main result of this research, and the inertial coefficient equations follow.

1. **For pipe above the bottom:**

$$C_M = 1.97 + 4.97\left(\frac{-Z}{L}\right)$$

gives "conservative estimates of the maximum forces" that are "quite appropriate."

2. **For bottom-laid pipe:**

$$C_M = 3.20 + 4.16\left(\frac{-Z}{L}\right)$$

gives "conservative estimates of the maximum forces" that "provide design values for the most vulnerable position."

3. **For half-buried pipe:**

$$C_M = 1.14 + 2.78\left(\frac{-Z}{L}\right)$$

gives "conservative estimates of the maximum forces" when the "entire volume of the pipe" is used in inertial force calculations and it is assumed "that pressure differences would penetrate the bed material at least to the bottom of the half-buried pipe."

4. **Pipe in an open trench:**

$$C_M = 1.0 + 1.4\left(\frac{-Z}{L}\right)$$

can be used for pipes in a trench of any shape.

The drag coefficients when plotted against Reynolds number formed definite trends, but were widely scattered. Drag forces were found to be important when $D^3/H\,L < 0.02$, where D = pipe diameter and H = wave height. It was concluded that in the range covered by these tests drag forces could be ignored when estimating design forces.

Chakrabarti, S. K. (1973). "Wave forces on submerged objects of symmetry," in Proceedings of ASCE, Journal of Waterways, Harbors and Coastal Engineering Division, vol 99, no. WW2, May 1973, pp 147-164.

The author collected existing data on wave forces on six shapes including horizontal cylinders and half cylinders. Closed form expressions for wave forces were obtained using the Froude-Krylov approach and assuming: linear stokes, irrotational wave theory, negligible drag effects, no free surface effects, no runup, objects smaller than the wavelength, and no flow inside the objects. Chakrabarti based his results regarding horizontal cylinders on the work by Schiller (1971).

The following expressions were derived.

1. **Horizontal cylinder:**

$$F_H = C_H \rho V \dot{u}_o$$

$$F_V = C_V \rho V \dot{v}_o$$

where F_H = horizontal wave force

F_V = vertical wave force

C_H = 2.10

C_V = 2.00

ρ = mass density of water

V = volume of object

$\dot{u}_o$ = horizontal water particle acceleration at center of object (in its absence)

$\dot{v}_o$ = vertical water particle acceleration at center of object (in its absence)

2. **Horizontal half cylinder:**

$$F_H = C_H \rho V \left[\dot{u}_o + S_3(kR)\sigma v_o\right]$$

$$F_V = C_V \rho V \left[\dot{v}_o + S_4(kR)\sigma u_o\right]$$

where C_H = 2.00

C_V = 1.00 for deep submergence

S_3, S_4 = functions of kR involving J_n

J_n = Bessel function of first kind of order n

k = 2π per wavelength

R = radius of object

σ = 2π per wave period

$\dot{v}_o$ = vertical water particle accleration at center of object (in its absence)

$\dot{u}_o$ = vertical water particle acceleration at center of object (in its absence)

The author cautioned that C_H and C_V should not be confused with inertia coefficients for use in the Morison equation, although the forms of the forces on the horizontal cylinders are similar to the inertia part of the Morison equation.

For a discussion of this article, see Yamamoto, Nath, and Slotta (1974a) in this Appendix.

Yamamoto, T., J. H. Nath, and L. S. Slotta (1974a). "Discussion of 'Wave forces on submerged objects of symmetry,'" in Proceedings of ASCE, Journal of the Waterways, Harbors and Coastal Engineering Division, vol 100, no. WW2, May 1974, pp 155-157.

The authors discuss Chakrabarti's conclusions regarding wave forces on horizontal cylinders. They questioned his assumption that lift and drag forces are negligible and his choices of horizontal force coefficient ($C_H = 2.0$) and vertical force coefficient ($C_V = 2.1$). They contend on the basis of their own theoretical and experimental work that:

1. The inertia coefficient C_I is the same in both the horizontal and vertical directions.

2. When the cylinder is on the bottom, $C_I = 3.29$ and approaches 2.0 as the cylinder is moved up.

3. When the cylinder is on the bottom, potential flow theory predicts that the lift coefficient C_L is 4.49.

4. C_L becomes a very large negative value if there is even a small gap between the cylinder and the bottom.

5. As the cylinder continues to be moved up, C_L rapidly goes to zero.

The authors recommend the design of pipelines using $C_I = 3.3$ to 4.5.

Denson, K. H., and M. S. Priest (1974). Effect of angle of incidence on wave forces on submerged pipelines, Preprint from ASCE National Meeting on Water Resources Engineering, Jun 21-25, 1974, Los Angeles.

The authors conducted wave tank tests on horizontal cylinders removed from the boundaries to determine the effect of angle of wave incidence on measured wave forces. Four angles of incidence were used, including 90 (the reference), 70, 50, and 30 degrees. Four wave heights were used, including 6.4 (spilling breaker), 4.9, 3.6, and 1.8 inches. The wave period was 2 seconds. Extreme wave forces of "lift, transverse positive drag, and transverse negative drag" were determined from taped records and calibration. For these tests the water depth was held constant at 12 inches; the pipe diameter, 2 inches.

A dimensional analysis for this study identified three pertinent parameters:

1. θ, the small angle between the wave direction and pipeline axis
2. F/F_{90}, the ratio of the extreme drag or lift force for the test θ to the corresponding drag or lift force for θ = 90 degrees
3. H/D, the ratio of wave height to water depth

The results were presented as curves of F/F_{90} versus θ for a range of H/D values. As θ increased from 30 to 90 degrees, force ratios F/F_{90}

1. increased regularly from 0.2 to 1.0 for lift from stable waves
2. increased from 0.4 to a peak of 1.1 at θ = 70 degrees, before returning to 1.0, for both positive and negative drag from stable waves
3. also went from 0.4 through a 1.1 peak at 70 degrees for positive drag from breakers
4. increased from 0.6 to 1.0 for negative drag from breakers
5. increased from 0.3 to 1.0 for lift from breakers

Grace, R. A. (1971a). The effects of clearance and orientation on wave induced forces on pipelines – results of laboratory experiments, University of Hawaii, James K. K. Look Laboratory, Technical Report No. 15. Honolulu, HI, Apr 1971.

The author reports on tests in a three-foot-deep wave flume. Both vertical and horizontal forces acting on a 3-inch-diameter model pipeline were measured for wave heights of about 1 foot and periods from 2 to 4 seconds. The angle between the cylinder axis and the wave orthogonals was increased from 30 to 90 degrees and the gap between the cylinder and the bottom from 1/32 to 1-1/8 inches to determine the effects of orientation and clearance of the pipeline on wave forces. It was found in the limited range of the test conditions that:

1. Horizontal force is virtually insensitive to clearance but decreases rapidly as the orientation of the pipeline relative to the wave fronts approaches perpendicular.

2. Vertical force decreases as clearance increases and as the orientation changes as above. The author recommended a more comprehensive set of tests like those above as well as tests on pipelines in larger wave tanks or at sea to obtain data at higher Reynolds numbers.

Grace, R. A. (1971b). Submarine pipeline design against wave action, Look Laboratory, Hawaii, vol 2, no. 2, Apr 1971, pp 3-7.

The author asserts that force data and coefficients obtained from wave force field tests of vertical cylinders or steady flow model tests on horizontal cylinders cannot be directly applied to estimates of wave forces on submarine pipelines. He suggests that actual ocean experiments of high Reynolds number laboratory tests are required for the proper design of submarine pipelines in waves. He grants that model pipeline tests can provide some information for the designer in indicating vague trends in the real-life situation.

The author summarizes the results of his tests and those of others (1961 to 1971) on model pipelines in both steady flow and waves. For steady flow perpendicular to the pipeline in the Reynolds number range from 3×10^4 to 2×10^6, the drag coefficients varied from 0.4 to 1.6 and the lift coefficients from 0.3 to 1.3. For waves perpendicular to the pipeline, the drag coefficients varied from 1.0 to 3.4 and the lift coefficients from 2.0 to 8.5.

The effect of bottom clearance under the pipe and the angle between the pipe and the wave orthogonals (orientation) was presented in the form of normalized curves taken from the author's previous work. These curves illustrate that the horizontal force and lift are maximum for a pipe resting on the bottom perpendicular to the wave orthogonals, and both the horizontal force and lift decrease as the clearance increases and as the angle of orientation decreases.

The author made the following suggestions for design wave force coefficients. He suggested an inertia coefficient of 2.5, a drag coefficient of 2.0, and a lift coefficient of 3.0.

Grace, R. A. (1973). Available data for the design of unburied, submarine pipelines to withstand wave action, Australian Conference on Coastal Engineering, Sydney, Australia, May 14-17, 1973, Published by Institute of Engineering, Sydney, Australia, National Conference Publication n 73/1, 1973, pp 59-66.

The author suggests methods for the computation of maximum wave forces on pipelines. He considers pipelines lying directly on or just above the seafloor laid at orientations of 30 to 90 degrees to the direction of the waves. Breaking wave forces are not addressed. Some of the design problems of trenched, ballasted, and buried pipelines are mentioned, but these are not discussed in detail.

The author suggests for design that the Airy wave theory be used to predict wave particle velocities but that the wave particle accelerations predicted by this theory be increased by a factor of 1.5. He expresses concern that force coefficients (which he calls C_D', C_I', etc.) are derived using theoretical kinematics and measured forces, but that these same coefficients are used as design coefficients (which he calls C_D, C_I, etc.) for pipelines to operate in actual kinematic conditions. He has no cure but cautions that the same wave theory as the one used in force coefficient determination should be used for design.

Based on previous data taken by others and his own previous work, the author suggests a procedure for determining force coefficients for pipelines laid perpendicular to the wave using the dimensionless parameter

$$\beta = \frac{\pi}{4}\left(\frac{C_I'}{C_D'}\right)\left(\frac{1}{S_o/D}\right)$$

where S_o = amplitude of water particle motion

D = pipe diameter

With this parameter the horizontal inertial coefficient $_*C_I'$, drag coefficient $_*C_D'$, vertical inertial coefficient $_*C_W'$, and lift coefficient $_*C_L'$ can be determined by trial and error using a table of "reference coefficients" defined below.

The foregoing force coefficients, $_*C_I'$, etc., are reference coefficients for the worst case (pipelines laid perpendicular to the wave orthogonals). The author provides for the correction of these coefficients to account for the height of the pipeline above the bottom, h, and the orientation relative to the wave orthogonal, α. He plots the relative clearance, h/D, and orientation, α, versus the ratio of horizontal force coefficient to the reference coefficient, Ω, and the ratio of the vertical force coefficient to the reference coefficient, λ. For example,

$$\Omega_D = C_D/_*C_D \quad \text{and} \quad \lambda_L = C_L/_*C_L$$

These ratios are plotted by classes of pipeline conditions based on the range of the dimensionless parameter, β.

For the computation of horizontal forces the Morison equation is suggested, using the values of $_*C_D'$ and $_*C_I'$ derived above. For vertical forces the following equation is suggested using the values of $_*C_L'$ and $_*C_W'$ derived above.

$$P = C_L'\left(\frac{\rho}{2}\right) D\, \ell\, u_{max}^2 \left|\sin \sigma \tau\right| \sin \sigma \tau + C_W' \rho \left[\left(\frac{\pi D^2}{4}\right) \ell\right] u_{max}\, \sigma \cos \sigma \tau$$

where P = vertical force

ρ = mass density of water

ℓ = length of pipe considered

u = Airy wave particle horizontal velocity

σ = $2\pi/T$

τ = time

Johansson, B., and E. Reinius (1963). "Wave forces acting on a pipe at the bottom of the sea," in Proceedings of Tenth International Association of Hydraulic Research Congress (Paper 1.7). London, England, vol 1, 1963, pp 47-52.

The authors conducted wave tank tests on a 50-mm-diameter model pipeline on the floor of the tank at water depths of 350, 450, and 550 mm. Wave periods of 8.9 seconds and heights from 153 to 233 mm were used. Five tests were run with the pipe perpendicular (90 degrees) to the wave direction, two each with the pipe at 40 and 20 degrees to the wave direction, and one test with the pipe parallel to the wave direction. It was assumed that both the horizontal and vertical wave forces were a function of the square of the horizontal water particle velocity at the center of the pipe, u^2, as determined using the first order theory of progressive waves, that is,

$$P_H = C_H D \frac{u^2}{2g} \gamma$$

$$P_V = C_V D \frac{u^2}{2g} \gamma$$

where P_H = horizontal force

P_V = vertical force

C_H = horizontal force coefficient

C_V = vertical force coefficient

D = diameter of pipe

γ = specific gravity

The average force coefficients determined from the five 90-degree tests were $C_H = 3.7$ and $C_V = 3.4$.

Johnson, R. E. (1970). "Regression model of wave forces on ocean outfalls," in Proceedings of ASCE, Journal of Waterways and Harbors Division, vol 96, no. WW2, May 1970, pp 289-305.

The author performed wave tank tests on a 4.5-inch-diameter cylinder which rested on the bottom of the tank. Waves varying in height from 0.99 to 9.27 inches were formed in water depths from 5 to 14 inches. A regression analysis which intentionally omitted wavelength and celerity was used in the analysis of the maximum wave force data. The data and the regression analysis prediction were plotted $F/\rho g D^3$ against H/D for 10 values of h/D between 1.11 and 3.11 where: F = wave force, ρ = mass density of fluid, D = diameter of cylinder, H = wave height, and h = water depth. It was concluded that wave forces on underwater structures can be predicted using regression analysis.

A discussion of this article can be found under Petrauskas (1971) of this Appendix.

Petrauskas, C. (1971). "Discussion of 'Regression model of wave forces on ocean outfalls,' by R. E. Johnson," in Proceedings of ASCE, Journal of Waterways and Harbors Division, vol 97, no. WW2, May 1971, pp 414-417.

The author compared Johnson's data with wave force predictions based on the Morison equation using $C_I = 2.0$ and $C_D = 1.0$. Petrauskas found that his predictions compared well with the original author's data and predictions except for the tests in which h/D values were less than 1.55. Petrauskas alleged that free surface effects caused this limit on his good correlation. He disagrees with Johnson's contention that wavelength is not significant in calculating maximum force. Although he grants that wavelength is not significant if the drag force dominates over inertial force, he notes that in the range of Johnson's experiments there are significant inertial forces.

Keulegan, G. H., and L. H. Carpenter (1958). "Forces on cylinders and plates in an oscillating fluid," Journal of Research of the National Bureau of Standards, vol 60, no. 5, May 1958, pp 423-440. (Research Paper 2857)

This paper is a major achievement in the study of wave forces acting on horizontal cylinders. It is here that the time dependency of cylinder added-mass and damping coefficient is first demonstrated.

For a cylinder in a field of sinusoidal motion — where the velocity is given by

$$U = -U_m \cos \sigma t$$

(U_m being the semiamplitude of the current and $\sigma = 2\pi/T$) — the authors show that:

$$\frac{F}{\rho U_m^2 D} = f\left(\frac{t}{T}, \frac{U_m t}{D}, \frac{U_m D}{\nu}\right)$$

F is the force on the cylinder per unit length, where $U_m D/\nu$ is a Reynolds number and $U_m T/D$ is termed the "period parameter." The authors then develop a fundamental relationship for the nondimensionalized force which is shown to be a form of the Morison equation. This force equation is time dependent and contains a parameter, ΔR, which is referred to as a remainder function.

Experiments were conducted in a standing wave tank having a length of 242 cm and a depth of 70 cm. Test objects (including cylinders with diameters from 0.5 to 3.0 inches) were placed 25 cm below the water surface in the tank at midsection. Expressions for computing water particle motion — needed in evaluation of the inertia coefficient, C_m, and the drag coefficient, C_d, from the measured force data — were derived for the wave tank test section.

It was determined that the period parameter was a useful dependent variable for plotting C_m and C_d values computed from the measured force data. C_m and C_d achieve minimum and maximum values, respectively, for a period parameter of around 15. It is hypothesized that when $U_m T/D = 15$, a single vortex is formed for each half cycle of sinusoidal fluid motion. This fact is supported by photographs taken during the tests. The authors conclude that eddy shedding has a significant effect on variations of the inertia and drag coefficients.

For those test conditions in which it can be reasoned that C_m and C_d do not have the same constant values at all phases of the wave cycle, the authors computed the remainder force function, ΔR. Once a curve of ΔR as a function of wave phase θ, was obtained, it was possible to plot curves of variable $C_m(\theta)$ and $C_d(\theta)$. Maximum variability of these coefficients occurred for $U_m T/D$ of about 15. C_m, for example, varied between a value of -2 and 2 with a cyclic period twice that of the wave period.

Muellenhoff, W. P., and L. S. Slotta (1971). "Investigation of the forces on a submerged cylinder due to surface water wave," in Proceedings IEE Conference, Engineering in the Ocean Environment, San Diego, CA, Sep 21-24, 1971, pp 58-63.

The authors measured horizontal and vertical wave forces on a 5.5-inch-diameter 3-foot-long horizontal cylinder. The cylinder was placed on and near the floor of a foot-deep wave basin perpendicular to the direction of the waves. Waves ranging in height from 0.5 to 3.5 inches, with periods of 0.5 to 1.5 seconds, and lengths of 10 to 80 inches were used in the tests. These tests were conducted to provide the basis for planned field tests of ocean bottom mounted pipes.

It was anticipated before the tests that the inertial components of the forces would be linearly proportional to H/T^2 (H = wave height, T = wave period), and that these inertial forces would be much greater than the drag forces. Both these facts were borne out by the tests. When the measured forces were plotted against H/T^2, straight lines of data points for narrow ranges of wavelength resulted, and it was found that the inertial forces were 3 to 5 times the drag forces. The wide scatter noted when the wave forces were plotted against H/D (D = cylinder diameter) was further evidence of the fact that wave forces are wave period dependent.

Priest, M. S. (1971). "Wave forces on exposed pipelines on the ocean bed," in Proceedings of Third Offshore Technology Conference, vol 1, Apr 1971, pp 549-552.

The author questions the application of either "steady-state" force coefficient formulas or the Morison equation to the design of submarine pipelines. He performed a dimensional analysis of the problem of a shallow water solitary wave on a pipeline normal to wave incidence. He considered the parameters of stillwater depth, **D**, pipe diameter, **d**, wave height, **H**, and specific weight of water, γ. He ignored wave period and length and did not consider force coefficients. He plotted two sets of horizontal and vertical wave force data which were measured by others. These plots of $P^*/\gamma D d$ against **H/D** were fitted by eye, and these equations were derived.

$$P_H = 0.18\,\gamma D d \left(\frac{H}{D}\right)^{1.63}$$

$$P_V = 0.16\,\gamma D d \left(\frac{H}{D}\right)^{1.56}$$

Rance, P. J. (1969b). "The influence of Reynolds number on wave forces," in Proceedings of the Symposium on Research on Wave Action, Delft, The Netherlands, vol 4, Jul 1969. (P r 13)

The author reports the results of experiments at the Hydraulic Research Station, Wallingford, England, where forces were measured on test cylinders with diameters ranging from 0.025 meter to 0.3 meter. A pulsating water tunnel having a test section 2.3 meters high by 0.5 meter wide was used in the experiments. This facility was capable of generating oscillatory flow having a semiorbit range of 0 to 2.5 meters and a period range of 4 to 14 seconds.

Contrary to the findings of Keulegan and Carpenter (1958), the author found that the force acting upon test cylinders was dependent upon the Reynolds number. A nondimensional force parameter, $F T^2/\rho D^3$, was plotted against the dimensionless parameter **a/D** (where F = force, T = period, ρ = density of water, D = diameter of the cylinder and **a** = semiorbit length). This latter parameter is, effectively, the same as the period parameter discussed by Keulegan and Carpenter. The author clearly demonstrated a Reynolds number dependency: with separate traces of the force parameter against **a/D** for different flow Reynolds numbers.

At low Reynolds numbers (i.e., around 10^4) the transverse forces were found to be of the same order of magnitude as the in-line forces. The transverse forces, which are due to vortex shedding, had a frequency, n, corresponding to a Strouhal number S = n D/V (where V is the velocity) of about 0.2 ±6%. There was no variation in this frequency with a/D or with the Reynolds number.

Generally, the magnitude of the high-frequency in-line forces (also caused by vortex shedding) were low (less than 10%) compared to the main in-line force. They were found to have no definite frequency.

Estimates of the inertial force coefficient made from the main in-line force curve indicated a value of 2.0 (the same value derived from potential flow theory for a cylinder in uniformly accelerated flow).

The author suggests that the earlier findings of Keulegan and Carpenter, that the forces were not dependent on the Reynolds number, may have resulted from limitations of the experimental conditions. The test cylinders were of a significant size compared with the water depth that it was possible that there was a Froude number effect

In a verbal discussion at the symposium Rance suggested that the transition range for forces which are predominantly inertial to those which are predominately drag depends upon the **a/D** ratio. The inertia and drag forces are equally important for an **a/D** ratio of about 1.5.

When asked if this transition region (which was indicated by Keulegan and Carpenter's results to be between **a/D** = 0.5 to **a/D** = 5) depends greatly on the Reynolds number and if it extends or becomes narrower for high Reynolds numbers, the author replied that his results did not go down below an **a/D** ratio of 1.5. He had performed a "rough analysis," however, and if one was prepared to accept an error of something like 10% of the total force, then an **a/D** ratio as low as 0.25 has to be selected. Rance concludes that the **a/D** ratio is far less important for higher Reynolds numbers.

Sarpkaya, T. (1975). "Forces on cylinders and spheres in a sinusoidally oscillating fluid," Transactions of AMSE, Journal of Applied Mechanics, paper presented at Applied Mechanics Western Conference, University of Hawaii, Honolulu, HI, Mar 25-27, 1975, pp 32-37.

The author conducted experiments with oscillatory flow past cylinders and spheres in a U-shaped vertical water tunnel. Circular cylinders of six diameters (6.35, 5.08, 4.45, 3.81, 3.18, and 2.54 cm) were used with one period of oscillation (T = 2.86 sec). Forces on the cylinder in line with and transverse to the direction of oscillation and the displacements and accelerations were recorded.

The data from the in-line force records were reduced based on a Fourier analysis using the Morison equation to determine the drag and inertial coefficients, C_D and C_M. These coefficients were found to be functions of U_m T/D (where U_m = maximum velocity of the fluid and D = cylinder diameter). Keulegan and Carpenter (1958) first noted this relationship of C_D and C_M with U_m T/D, which they called the "period parameter." The resulting curve of C_D versus the period parameter from Sarpkaya's work, which shows a maximum C_D of 2.3 at a period parameter of 12, compares very well with the work of Keulegan and Carpenter. The curve of C_M versus the period parameter from Sarpkaya's work, which shows a decrease of C_M from 2.2 at a period parameter of 2 to a minimum value of 0.7 at a period parameter of 12, does not compare well with Keulegan and Carpenter.

The data from the transverse force records were reduced using the standard lift equation:

$$F_L = \frac{C_L}{2} \rho U_m^2 D L$$

where C_L = coefficient of lift

ρ = mass density of water

L = length of cylinder

A plot of C_L versus the period parameter shows a maximum C_L of 3 at a period parameter of 17 with a smaller peak of 2.8 at a period parameter of 10.

The author's tests were in the range of period parameters from 2 to 50. He found no correlation between Reynolds number and the force coefficients (C_D, C_M, and C_L). But he noted the existence of a unique relationship between C_D and C_M. He also noted that the transverse force on the cylinder is as large as or larger than, the in-line force and that at large values of period parameter the inertial force is a small part of the total in-line force.

Schiller, F. C. (1971). Wave forces on a submerged horizontal cylinder, United States Naval Postgraduate School, Report No. AD727691, Monterey, CA, Jun 1971.

The author reports the results of experiments on wave forces on a rigid horizontal circular cylinder located near a plane bottom boundary. All data was obtained in a wave tank having a rectangular cross section 2 feet deep with a width of 15 inches. Except for one test series when the smooth plexiglass cylinder was positioned one radius above the tank floor, most of the tests were conducted with the cylinder suspended by wires approximately 1/8 inch off the bottom. In order to prevent a high-velocity flow beneath the cylinder, a flexible plastic curtain was draped from the cylinder to the bottom for those tests where the cylinder gap dimension was 1/8 inch.

The vertical wires supporting the cylinder as well as those attached to the cylinder fore and aft were connected to strain-gaged cantilever beams. These beams were used as vertical and horizontal force measuring transducers. Wave height measurements were made with a parallel wire resistance gage mounted approximately 5 feet in front of the cylinder.

The author concludes that for the wave heights tested the horizontal wave forces on the cylinder increased linearly with the wave height. When the cylinder was located on the bottom, the vertical force was much smaller than the horizontal. Except at very small wave amplitudes, the vertical wave force showed a nonlinear increase with wave amplitude.

An approximate analysis – based on linear wave theory – appeared to give good results for cases where the depth of submergence was large. It was important, however, to know the proper value for the added mass coefficient.

The author provides numerous examples of wave force traces for the cases investigated; and a complete compilation of measured data appears in his appendix. Some of these data have been used by Yamamoto, et al. (1973a) in their comprehensive report on wave forces on horizontal cylinders.

Yamamoto, T., J. H. Nath, and L. S. Slotta (1973a). "Yet another report on cylinder drag on wave forces on horizontal submerged cylinders," Oregon State University, Engineering Experiment Station, Bulletin No. 47. Corvallis, OR, Apr 1973*

This report begins with a thorough review of the literature on the effects of steady and accelerating flow on horizontal cylinders. The physics of the fluid flow, even for the case of steady flow, is complex. Besides the influence of Reynolds number, forces on cylinders in steady flow are affected by the proximity of the ground plane, free surface effects (drag force), the influence of the wall boundary layer (cylinder lift and drag forces), vortex formation near the wall, and fluid turbulence (drag force).

For the more complex case of accelerating flow, one must consider the effects of convective acceleration and boundary/cylinder interaction on inertial forces, nonsteady viscous effects on the drag force which include skin friction and the formation of vortices behind the cylinder, and viscous effects on the added mass.

The authors report the results of a series of wave tank experiments which were limited to test cases wherein one could neglect drag and the effects due to convective acceleration. Their data and previously published data by Schiller (1971) were used to calculate lift and inertia coefficients which agreed closely with those derived from potential flow theory.

Based upon their search of the technical literature and test results, the authors conclude:

1. That the lift force which acts upward when the pipeline rests on the seafloor and downward when the pipeline is suspended less than one pipe diameter above the seafloor indicate a pipe failure mode wherein the pipe is alternately lifted from and then dropped to the seafloor.

2. That near the seafloor the added mass of a cylinder is more than twice that of the same cylinder far removed from the seafloor.

3. That for steep waves in relatively shallow water, the neglecting of convective acceleration can amount to an error of more than 30% in the pipe inertial forces.

Among recommendations for future research, the authors suggest:

1. A more sophisticated review of the technical literature on steady state and unsteady flow effects on cylinders so that compact design aids can be developed.

2. An experimental investigation of the effect of convective accelerations on pipelines.

3. A study of the interaction of the bottom boundary layer with pipeline lift and drag forces.

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*See also Yamamoto, Nath, and Slotta (1974).

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LIST OF SYMBOLS

C_D	Drag coefficient
C_H	Horizontal force coefficient
C_I	Inertia coefficient
C_L	Lift coefficient
C_{LA}	Lift coefficient due to flow asymmetry
C_{LE}	Lift coefficient due to vortex shedding
C_V	Vertical force coefficient
C_W	Vertical inertia coefficient (after Grace (1971))
D	Diameter of cylinder or pipeline
F_D	Drag force
F_H	Horizontal wave force
F_{HD}	Horizontal drag force
F_{HI}	Horizontal inertia force
F_{HLA}	Horizontal lift force due to flow asymmetry
F_{HLE}	Horizontal lift force due to vortex shedding
F_I	Inertia force
F_L	Lift force
F_{LA}	Lift force due to flow asymmetry
F_{LE}	Lift force due to vortex shedding
F_V	Vertical wave force
F_{VD}	Vertical drag force
F_{VI}	Vertical inertia force
F_{VLA}	Vertical lift force due to flow asymmetry
F_{VLE}	Vertical lift force due to vortex shedding
H	Wave height
L	Wave length
L_o	Deep water wave length
N_R	Reynolds number
S	Strouhal number, $S = nD/V$
T	Wave period
U_m	Maximum wave-induced water particle velocity
V	Free stream velocity
d	Water depth
e	Gap between cylinder and bottom boundary
g	Acceleration of gravity
k	Wave number, $k = 2\pi/L$
n	Vortex shedding frequency (Hertz)
t	Time
u	Horizontal wave-induced water particle velocity
u_{max}	Maximum horizontal wave-induced water particle velocity
$\dot{u}$	Horizontal wave-induced water particle acceleration
$\dot{u}_{max}$	Maximum horizontal wave-induced water particle acceleration
v	Vertical wave-induced water particle velocity
v_{max}	Maximum vertical wave-induced water particle velocity
$\dot{v}$	Vertical wave-induced water particle acceleration
$\dot{v}_{max}$	Maximum vertical wave-induced water particle acceleration
x	Horizontal space coordinate
y	Vertical space coordinate
β_{HM}	Phase angle at which the sum of the horizontal inertia and drag forces is maximized
β_{VM}	Phase angle at which the sum of the vertical inertia and drag forces is maximized
η	Wave elevation
ν	Kinematic viscosity
ρ	Mass density of water
ω	Wave frequency (radians/sec)

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